# 5-3 Videos Guide

## 5-3a

### Theorem:

- Fundamental Theorem for Line Integrals: If $C$ is a smooth curve and $f$ is a differentiable function whose gradient $\nabla f$ is continuous on $C$, then
  $\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$, where $\mathbf{r}(t)$, $a \le t \le b$ describes $C$
  $= f(x_2, y_2, z_2) - f(x_1, y_1, z_1)$ (an analogous expression exists for the $\mathbb{R}^2$ case)

## 5-3b

- Description of path independence
  - $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ for any two paths $C_1$ and $C_2$ that connect the same two points

## 5-3c

### Theorems:

  - $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed paths $\Leftrightarrow \int_C \mathbf{F} \cdot d\mathbf{r}$ is path independent
    $\Longrightarrow$ $\mathbf{F}$ is a conservative vector field (this means there exists a potential function $f$ of $\mathbf{F}$)
    $\Longrightarrow \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

### Exercises:

## 5-3d

(a) Find a function $f$ such that $\mathbf{F} = \nabla f$ and (b) use part (a) to evaluate $\int^C \mathbf{F} \cdot d\mathbf{r}$ along the given curve $C$.

- $\mathbf{F}(x, y) = (3 + 2xy^2)\,\mathbf{i} + 2x^2y\,\mathbf{j}$,
  $C$ is the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $\left(4, \frac{1}{4}\right)$

## 5-3e

- $\mathbf{F}(x, y, z) = (y^2z + 2xz^2)\,\mathbf{i} + 2xyz\,\mathbf{j} + (xy^2 + 2x^2z)\,\mathbf{k}$,
  $C: x = \sqrt{t},\; y = t + 1,\; z = t^2,\; 0 \le t \le 1$

## 5-3f

- Law of Conservation of Energy